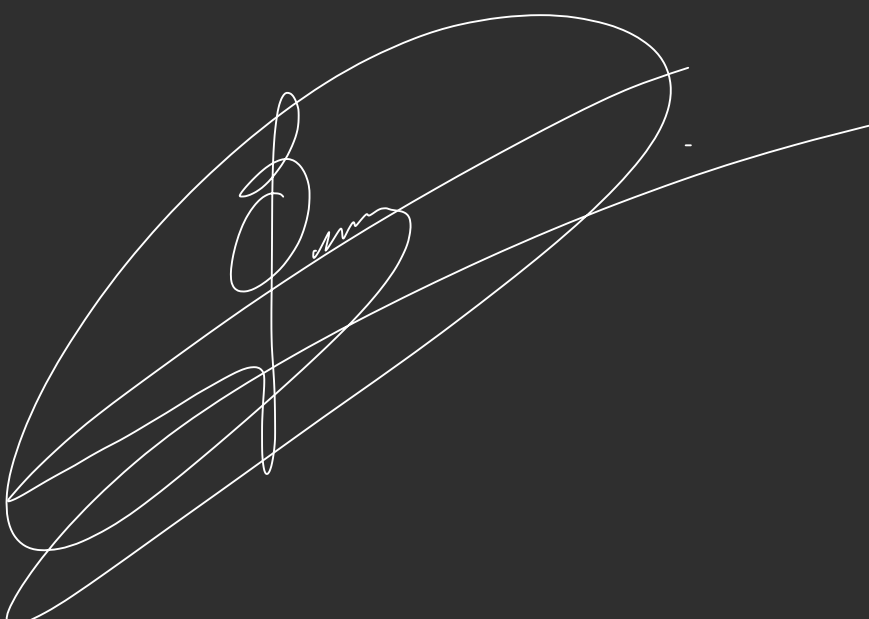
P.S. my cousin Sienna drew this so shoutout Sienna Jung. (she's 10 btw).

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Love from Jamie!



✱ Indices

A. Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

(a) $r + s \leqslant 0$,
(b) $s \leqslant 0$,
(c) $r \leqslant 0$,
(d) $r \geqslant s$.

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

instanly you should be thinking of how can I simplify this.

so

$$6^{r+s} = (2 \cdot 3)^{r+s}$$
$$= 2^{r+s} \cdot 3^{r+s}$$
$$12^{r-s} = 2^{r-s} \cdot (6^{r-s})$$
$$8^r = 2^{3r}$$
$$9^{r+2s} = 3^{r+2s} \cdot 3^{r+2s}$$

numerator : $2^{r+s} \cdot 3^{r+s} \times 2^{r-s} \cdot 2^{r-s} \cdot 3^{r-s}$

: $2^{r+s} \cdot 3^{r+s} \times 2^{2r-2s} \cdot 3^{r-s}$

: $2^{3r-s} \cdot 3^{2r}$

Denominator : $2^{3r} \times 3^{2r+4s}$ ∴ the fraction be comes

$$\frac{2^{3r} \cdot 2^{-s} \cdot 3^{2r}}{2^{3r} \cdot 3^{2r} \cdot 3^{4s}} = \frac{2^{-s}}{3^{4s}}$$

$$= \frac{1}{3^s \cdot 2^{4s}}$$

∴ $s \leq 0$ (B)

* trig / functions

B. The greatest value which the function

$$f(x) = \left(3\sin^2(10x + 11) - 7\right)^2$$

takes, as x varies over all real values, equals

(a) -9, (b) 16, (c) 49, (d) 100.

As $f(x) = (3\sin^2(10x+11) - 7)^2$

the greatest value of $\sin^2(a)$ where $a \in \mathbb{R}$ is 1

$\therefore f(x)_{max} = (-4)^2 = 16$

This is a common mistake people will make.

As $\sin^2(a)$ is always positive, $|3a - 7| \leq |-7|$

where $0 \leq a \leq 1$ $\therefore$ the true maximum of this function is when $\sin^2(10x+11)$ is 0

$\therefore f(x) = (-7)^2 = 49$

and $\therefore$ (C)

* Graph sketching / trig

C. The number of solutions x to the equation

$$7\sin x + 2\cos^2 x = 5,$$

in the range $0 \leqslant x < 2\pi$, is

(a) 1, (b) 2, (c) 3, (d) 4.

$7\sin x + 2\cos^2 x = 5$

For this specific question, algebra is faster.

$7\sin x + 2(1-\sin^2 x) = 5$

$2\sin^2 x - 7\sin x + 3 = 0$

$u = \sin x$

$2u^2 - 7u + 3 = 0$

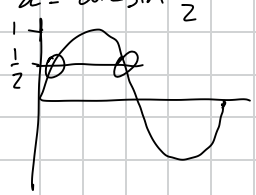
$(2u-1)(u-3) = 0$

$u = \frac{1}{2}$

$u = 3 \rightarrow$ not possible

$\sin x = \frac{1}{2}$

$x = \arcsin \frac{1}{2}$



2 solutions

* Circle Geometry

D. The point on the circle

$$(x-5)^2+(y-4)^2=4$$

which is closest to the circle

$$(x-1)^2+(y-1)^2=1$$

is

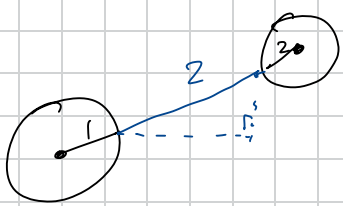
(a) $(3.4, 2.8)$, (b) $(3,4)$, (c) $(5,2)$, (d) $(3.8, 2.4)$.

$(x-5)^2+(y-4)^2=4$

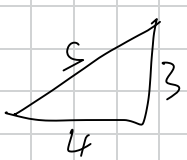
centre: $(5,4)$ $r=2$

$(x-1)^2+(y-1)^2=1$

centre: $(1,1)$ $r=1$



work out the distance between centres:



$\therefore (1,1)+\frac{3}{5}(4,3)$

$(1,1)+\frac{12}{5},\frac{9}{5}=$

$=\left(\frac{17}{5},\frac{14}{5}\right)=\left(3\frac{2}{5},2\frac{4}{5}\right)$

$=(3.4, 2.8)$

$\therefore$ (A)

E. If x and n are integers then

$$(1-x)^n (2-x)^{2n} (3-x)^{3n} (4-x)^{4n} (5-x)^{5n}$$

is

(a) negative when $n > 5$ and $x < 5$,
(b) negative when n is odd and $x > 5$,
(c) negative when n is a multiple of 3 and $x > 5$,
(d) negative when n is even and $x < 5$.

$f(x) = (1-x)^n (2-x)^{2n} (3-x)^{3n} (4-x)^{4n} (5-x)^{5n}$

we want $f(x)$ to be negative.

counter examples:

$n > 5 \quad x < 5$

let $x = 0$

let $n = 6$ not negative $\therefore$ false

n is odd $\quad x > 5$

let $x = 6$

let $n = 1$

$(5)(-4)^2 (-3)^3 (-2)^4 (-1)^5$

$\ominus \; + \; \ominus \; + \; -$

$+ \; + \; + \; -$

$\therefore$ -ve

this is always true.

$\therefore$ B

F. The equation

$$8^x + 4 = 4^x + 2^{x+2}$$

has

(a) no real solutions;
(b) one real solution;
(c) two real solutions;
(d) three real solutions.

$2^{3x} + 4 = 2^{2x} + 4 \cdot 2^x$

let $u = 2^x$

$u^3 - u^2 - 4u + 4 = 0$

notice when $u = 1$
this cubic $= 0$
$\therefore u = 1$ is a solution

$2^x = 1$

$u - 1 \overline{)u^3 - u^2 - 4u + 4}$

1	1	−1	−4	4
	1	1	0	−4
		0	−4	0

$u^2 - 4$

$(u+2)(u-2)$

$2^x \neq -2$
not possible

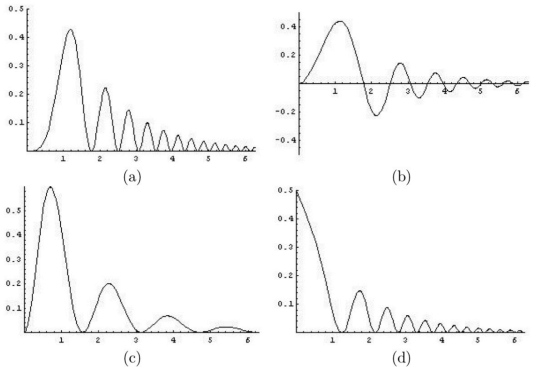
$2^x = 2$
$x = 1$
$x = 2$

$\therefore$ 2 solutions

(C)

G. On which of the axes below is a sketch of the graph

$$y = 2^{-x}\sin^2(x^2)?$$



$y = \frac{1}{2^x}\sin^2(x^2)$

$y > 0$ so we can eliminate (b)

when $x = 0$ $y = 0$ so we can eliminate (d)

when $y = 0$

$\sin^2(x^2) = 0$

$x^2 = 0, \pi, 2\pi, 3\pi \ldots$

$x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}$

so the frequency of $y = 2^{-x}\sin^2(x^2)$ intercepting the x-axis increases as x increases.

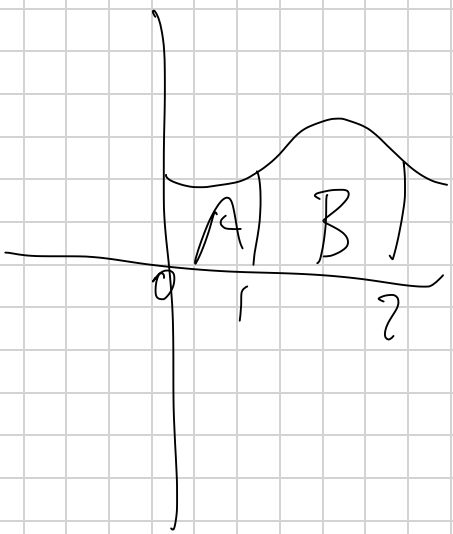
∴ the graph is (A)

H. Given a function $f(x)$, you are told that

$$\begin{aligned} \int_0^1 3f(x)\,\mathrm{d}x + \int_1^2 2f(x)\,\mathrm{d}x &= 7, \\ \int_0^2 f(x)\,\mathrm{d}x + \int_1^2 f(x)\,\mathrm{d}x &= 1. \end{aligned}$$

It follows that $\int_0^2 f(x)\,\mathrm{d}x$ equals

(a) -1, (b) 0, (c) $\frac{1}{2}$, (d) 2.



$3A + 2B = 7$

$A + B + B = 1$

$3A + 2B = 7$

$A + 2B = 1$

$2A = 6$

$A = 3$

$B = -1$

$A + B = 2$

(D)

I. Given that a and b are positive and

$$4\,(\log_{10} a)^2 + (\log_{10} b)^2 = 1,$$

then the greatest possible value of a is

(a) $\frac{1}{10}$, (b) 1, (c) $\sqrt{10}$, (d) $10^{\sqrt{2}}$.

$4(\log_{10} a)^2 + (\log_{10} b)^2 = 1$

$(\log_{10} b)^2$ is ≥ 0 $\forall\ b \in \mathbb{R}$

$\therefore \quad 4(\log_{10} a)^2 = 1$

$(\log_{10} a)^2 = \frac{1}{4}$

$\log_{10} a = \frac{1}{2}$

$a = \sqrt{10}$ //

(C)

J. The inequality

$$(n+1)+\left(n^4+2\right)+\left(n^9+3\right)+\left(n^{16}+4\right)+\cdots+\left(n^{10000}+100\right)>k$$

is true for all $n \geqslant 1$. It follows that

(a) $k<1300$,
(b) $k^2<101$,
(c) $k \geqslant 101^{10000}$,
(d) $k<5150$.

$(n+1)+(n^4+2)+(n^9+3)\ldots(n^{10000}+100)>k$

$\sum_{r=1}^{100} r$

$\frac{1}{2}n(n+1)$

$50(101) = 5050$

minimum is 5050

let $n=1$

$\sum_{r=1}^{100} 1 = 100$

$5150 > k$

$\therefore$ D