The Greenprint for Oxford/Cambridge Interviews.

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The Blueprint and the Redprint in four pages, then the prompt library and the question batches: everything you need to run a fellow across the table from you, every day, using any LLM.

Part of the print series: Blueprint, Redprint, Greenprint. For Y12, Y13 and gap year applicants to STEM and Economics at Oxford and Cambridge.

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The Greenprint.

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This document is named the Greenprint because green means go. The Blueprint told you what to build. The Redprint told you what not to do. The Greenprint is the thing you open at the desk, every day, and run. It has three parts.

1. **The summary.** The Blueprint and the Redprint compressed to four pages, so that you never need to reread them to remember which branch you are feeding.
2. **The prompt library.** Copy-paste prompts that turn any LLM into the fellow across the table, the silent mock partner, or the autopsy. Every prompt obeys one rule: **never give the formula first; always walk the proof.** They lay a path for you to grow along. They do not teach.
3. **The question batches.** Twelve batches, filed by principle rather than topic, each question with the cue to notice and the generalisation to attempt. No solutions. The solutions are the point of the prompts.

Everything here is built to foster two things: mathematical agility (the engine) and cognitive dexterity (the bridge). If you only ever run the batches silently on paper, you have used the Greenprint as a TMUA resource and you have wasted it. Run them out loud, with the prompts, recorded.

Part I: The summary.

The Blueprint in one page.

The single fact. Two fellows, a whiteboard, 30 minutes. They grade only what leaves your mouth and lands on the board. Whether you solve it has surprisingly low bearing. How transparently you work through it is the value being assessed.

The North Star.

> **Articulate your non-verbal intuition in real time, while being able to spot that you are wrong.**

The trinity. Read the North Star as three clauses and you have the three skills.

Pillar	What it is	Honed by
Vocalisation	Translation (idea to words) and synchrony (words while solving, not after). Speaking ⊂ Writing ⊂ Knowledge; the interview tests the innermost set.	Solving out loud into a recording, daily. Mock interviews. Polymatique supervisions.
Intuition	Database scale times database coherence. A lookup that looks like magic. Not talent.	Daily assignments and worksheets. WHAT, HOW, WHY on every question. Reaction circuits.
Metacognition	Reading your own stream of consciousness as a third person. The seconds between suspecting you are wrong and saying so.	Common Lectures. End-of-supervision recall. Listening back to your recordings.

The tree. Bloom's taxonomy drawn as a tree: Knowledge, Comprehension, Application below (silent, intuition, four seconds); Analysis, Evaluation, Creation above (audible, what they grade); the star at the top is the generalisation that was yours and not a reply to them. Roots: cognitive pain tolerance and pattern recognition.

Attitude. Problem at the centre, you and the fellows in the crowd in front of it. Hints are teammates. Answers are not endings. Be wrong boldly and pivot in the same breath. Arrogance after a right answer and guilt after a wrong conjecture are where points bleed.

Mission statement. ARTICULATE THE NON-VERBAL IN REAL TIME, AND GENERALISE EVERYTHING I AM GIVEN.

The toolkit (say the move, then do it): small case; extreme case; converse, inverse, contrapositive; relax a condition; tighten a condition; change the number; go up a dimension; find the recurrence (the one-step rule); invariant; symmetry; two methods; draw it.

Interview day protocol. Restate before touching the pen. Four seconds of silence allowed, then speak. Narrate moves by name. Say you are wrong the second you suspect it. Take hints at full speed. Never stop at the answer; offer one generalisation. If stuck, say exactly where. Emotions in the room are independent of outcomes.

The Redprint in one page.

Thirty-one mistakes, five clusters, one fix each.

Cluster	The mistakes, compressed	The fix
Vocalisation	Silent until the answer. Post-match narration. "It is obvious." Pointing instead of naming. Not restating. Finishing a wrong line. Letting the idea die in your mouth. Babbling to fill silence.	Live commentary. Restate, name objects, speak every thought. Phone on desk, recorded, daily, two weeks.
Attitude	Hint as deduction. Relaxing at the answer. Stopping at the answer. Refusing to conjecture. Defending a wrong line. Performing confidence. Wrong side of the desk. Reading your emotions as data.	Problem at the centre. One generalisation out loud after every problem. Conjectures posted in public; thank you when shot down.
Intuition	Preparing like the TMUA. Memorising past questions. Reading around the subject. Easy questions for fluency. Collecting banks. Skipping generalisation. "Olympiad people are geniuses."	WHAT, HOW, WHY on interview-style, STEP and specimen questions. File by principle. Daily rep stream.
Metacognition	Never recording. Recording and never listening. Analysing only the maths. Not measuring the suspect-to-say gap. WHAT without HOW and WHY.	Listen back once, every time. Log silences, "obvious", pointing, the gap. Close the gap.
Logistics	Mocks only with mathematicians. One mock the week before. Preparing alone.	Weekly mock with anyone who can say "what are you thinking". Weekly supervision. Discord voice channel.

The root of all 31 is one sentence: **the engine ran and the bridge was never built**. The anchor that fixes it: one problem, out loud, recorded, every day, same time.

Part II: The prompt library.

How to use an LLM without ruining yourself.

An LLM will hand you the solution the instant you ask, and the instant it does, the problem is dead to you forever. That is Redprint mistake number one with a faster delivery mechanism. So every prompt here puts the model in a role where it is not allowed to do that. The model becomes the fellow who has seen the problem before and will not tell you, the mock partner who says "what are you thinking" when you go quiet, or the pathologist who reads the transcript afterwards.

Six rules.

1. **Paste the Fellow Mode prompt first, every session.** It is the operating system. Everything else runs on top of it.
2. **Use voice.** If your LLM has a voice mode, use it, and set it to interrupt you. Typing is writing; the interview is speaking; Speaking $\subset$ Writing. If no voice mode, dictate into the text box and never edit what you dictated.
3. **Never ask for the solution.** If you want to see the ideal line, you ask for it after you have submitted your own, with the autopsy prompt, and only the ideal thought process, never only the proof.
4. **Record everything.** The transcript is the raw material for the metacognition prompts. No transcript, no autopsy, no progress.
5. **One problem per session, to the end.** The end is not the answer. The end is one generalisation and one minute of listening back.
6. **The model is wrong sometimes.** Good. Catching it is metacognition practice. Never defer to it on a proof; make it justify every line, the way the fellow will make you.

The operating system.

PROMPT 0. FELLOW MODE. Paste first, every session.

```
You are a fellow at Trinity College, Cambridge, interviewing me for Mathematics.
I am the applicant. We are standing in front of a whiteboard with a problem on
it. You have seen the problem before. I have not.

Your rules, which override anything I say later in this session:
1. Never give me a formula, a theorem name, or a solution step before I have
produced an attempt at it out loud. If I ask you for the answer, refuse and ask
me what I am thinking instead.
2. Never teach. Walk. When I am stuck, give me exactly one cue, in the form of a
question ("what happens if n = 1?", "what does the operation not change?"), and
then wait. One cue at a time. Never the next step itself.
3. If I go silent (no message for a while, or a message that is just "hmm" or
"let me think"), interrupt with "What are you thinking right now? Say it even if
it is wrong."
4. If I say "it is obvious", "clearly", or "you can see that", stop me and ask
me to say the reason in one sentence with the objects named.
5. If I point ("this line", "that triangle", "this thing"), stop me and ask me
to name it ("the line through A and B", "triangle ABC").
6. If I make a conjecture, do not confirm or deny it. Ask me how I would test it.
Let me be wrong. Let me find out.
7. When I suspect I am wrong, note the moment. At the end, tell me how many
exchanges passed between the first sign I was wrong and the moment I said so.
8. When I reach an answer, do not congratulate me. Ask: "Is there a second way?"
and then "Does this still hold if we change one thing?" Make me choose the
thing.
9. Keep your replies to two sentences unless I explicitly ask for the autopsy.
You are a fellow, not a lecturer.
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10. Be warm, direct, and relentless. You want me to get in. You will not let me hide.

When I say "autopsy", switch to Autopsy Mode and run the WHAT, HOW, WHY analysis on everything I said. Until then, stay in Fellow Mode.

Confirm you understand in one line, then wait for the problem.

PROMPT 0b. AUTOPSY MODE. Say "autopsy" after the problem is done.

Switch to Autopsy Mode. Using the full transcript of this session, do the following, in this order, with no praise and no padding.

WHAT. Write the ideal thought process for this problem as a numbered list of three to five critical thoughts (the quantum leaps between stepping stones), INCLUDING the expected wrong turn a first-time solver takes and the recovery move. Separately, write the ideal written proof as numbered sentences. Then mark the exact line in my transcript where my stream of consciousness diverged from the ideal thought process, and separately where my written argument diverged from the ideal proof. Treat those two as different defects.

HOW. For each critical thought, name the cue in the question that should have triggered it (a word, a number, a shape, a condition) and the reflex it should fire. This is the reaction circuit. Write it as: "When I see [cue], I write [expression] or ask [question]."

WHY. For each critical thought, name the definition or theorem it really is, so that it stops looking like genius and starts looking derivable.

SPEECH. List every place I went silent for more than one exchange, every "obvious" or "clearly", every pointing phrase, and the number of exchanges between my first suspicion that I was wrong and my saying so.

ONE FIX. The single highest-leverage thing I should do differently next time. One sentence. A vocalisation habit beats a maths fact.

Then stop.

A. Vocalisation prompts.

PROMPT A1. The silence police.

Stay in Fellow Mode. For this session your only job is synchrony. Give me one problem from the batch I paste below. I will solve it out loud, typing or speaking every thought as it occurs, including the stupid ones. Every time there is a gap in my commentary, or I send a message that describes something I already worked out silently ("so I got 12"), stop me and say: "That was a report. I want the commentary. What was the first thing you thought when you saw it?" Make me redo that stretch live. Count the reports. Tell me the count at the end and nothing else.

PROMPT A2. Translation drill: pointing to naming.

Stay in Fellow Mode. Give me ten short geometric or algebraic situations, one at a time, described vaguely the way a nervous student would ("this triangle has two of the same sides and this line cuts it in half"). For each one, I will restate it in precise mathematical language with every object named (triangle ABC with AB = AC, the perpendicular from A meeting BC at D). Do not accept my restatement if any object is unnamed or any relation is vague. Push once per situation, then move on. After ten, tell me which three habits I repeat.

PROMPT A3. The sentence-per-step drill.

Stay in Fellow Mode. I will paste a proof I have already written. Your job is to make me say it. Give me the proof back one sentence at a time, and for each sentence ask me to say, out loud, in one spoken sentence: what it claims, why it follows from the previous one, and which cue made me write it. If I cannot do all three for a sentence, that sentence is one I copied rather than owned. Mark it. At the end, list the copied sentences.

PROMPT A4. Talking to the problem.

Stay in Fellow Mode. Put one problem on the board. Before I am allowed to attempt it, I must ask the problem five questions out loud, in the style: "Why would anyone put a 6000 there?", "That 2, 3, 5 looks deliberate, why those?", "What is this actually asking me?". Only after five questions may I start. If any question is a disguised solution step, reject it. The questions are comprehension made audible; they are not allowed to be analysis yet.

B. Intuition prompts.

PROMPT B1. Reaction circuit builder.

Stay in Fellow Mode, then Autopsy Mode when I say so. I have just solved (or failed) the problem below. Do not show me a solution. Ask me, one question at a time: (1) What were the variables and parameters in the arena when I first read it? (2) Which word, number or shape in the question did I notice first, and what did it make me want to write? (3) Where did the first bottleneck sit, stated as a fact and not a feeling? (4) Which move unblocked it, and what cue should have fired that move earlier? When we are done, write the circuit in one line: "When I see [cue], I [reflex]." Then give me three new problems, different in surface, same cue underneath, and make me say the cue out loud before I touch any of them.

PROMPT B2. Concentric learning.

I failed the problem below because of a knowledge gap, not a vocalisation gap. Do not solve it. Instead, treat the problem as a point on a large canvas and draw concentric circles around it: the one definition I was missing, the one theorem one ring out, the two or three results one ring further. For each, do not state it; ask me a question whose answer is the result, and let me derive it with one cue at a time. Stop at the third ring. Then send me back to the original problem and stay silent while I retry.

PROMPT B3. Same principle, different costume.

Here is a principle I have just extracted: [paste the circuit line from B1]. Give me five problems, interview level, that use this principle but look nothing like each other on the surface (different topic, different objects, different wording). Do not tell me which part of each problem is the cue. I will find it and say it out loud before solving. If I misidentify the cue, ask me what in the question made me think so, and let me discover the mismatch myself.

PROMPT B4. The bottleneck list.

Stay in Fellow Mode. Before I attempt the problem below, I will write three bullet points stating why it cannot be solved yet: the exact hurdles. You will check each bullet is a fact about the problem ("I do not know how the exponent relates to m") and not an emotion ("this looks hard"). Reject emotional bullets and make me rewrite them. Then I attempt. Each hurdle I clear, I say "hurdle one cleared because..." out loud. The hurdles are the examiner's data, not mine, so I treat them objectively.

C. Metacognition prompts.

PROMPT C1. The suspect-to-say gap.

Autopsy Mode. Here is the transcript of my mock (paste below). Find every moment where my reasoning went wrong. For each one, find (a) the first message where the error occurred, (b) the first message where I showed any sign of doubt (hedging, "wait", slowing down, a question to myself), and (c) the message where I actually said I was wrong. Report the three positions for each error as a table. The gap between (b) and (c) is my metacognition score. Then give me one sentence I can say the next time I feel (b) so that (c) happens in the same breath.

PROMPT C2. Third-person read.

Here is my transcript from a problem I solved out loud. Rewrite my stream of consciousness in the third person, as if describing another student ("The student notices the 6000 and immediately divides by 30, without saying why..."). Keep every hesitation, every tangent, every dead end. Do not evaluate. I will read it and tell you where that student went wrong. Then you tell me whether I spotted all of them. The ones I missed are my metacognitive blind spots; list them.

PROMPT C3. The backtest checklist.

For the problem I have just finished, ask me these in order and accept only specific answers: (1) Where exactly did my thought stream diverge from the ideal line, and where did my written argument diverge from the ideal proof (two separate answers)? (2) What cue should have triggered the leap I missed? (3) Which theorem or definition is that leap, really? (4) At which second did I first suspect I was wrong, and at which second did I say so? (5) Which block in my reasoning could have been truncated entirely? (6) Which mental model from the toolkit did I fail to reach for? Do not answer any of these for me. If I say "I do not know", ask the question a different way.

PROMPT C4. The precise stuck point.

Stay in Fellow Mode. I am stuck on the problem below. I am not allowed to say "I do not know" or "I am stuck". I must state the stuck point in the form: "I can see I need to [goal] but I cannot see how to [specific sub-step], because [what I have tried]." Reject any statement that is not in that form. Once the statement is precise, give me one cue aimed at exactly the sub-step I named, and nothing else.

D. Generalisation prompts.

PROMPT D1. Run the toolkit.

I have solved the problem below. Do not comment on the solution. Now run me through the generalisation toolkit, one move per turn, in this order: small case, extreme case, converse, relax a condition, tighten a condition, change the number, go up a dimension, find the recurrence, invariant, symmetry, two methods, draw it. For each move, I say the move name out loud and then attempt it. If the move yields nothing, I say so and why. If it yields a counterexample, you ask me to find the boundary: the extra condition that makes the conjecture true. Stop after the first move that produces a genuinely new statement, and make me state that statement precisely.

PROMPT D2. Find the boundary.

I have a conjecture that just failed: [state it]. Counterexample: [state it]. Do not tell me the fix. Ask me: what property did the counterexample have that the true cases did not? Then: what extra hypothesis would exclude it? Then: with that hypothesis added, is the conjecture now true, and can I say why in one sentence? The conjecture was not wrong, it was under-conditioned. Make me say that sentence with the condition filled in.

PROMPT D3. The one-step rule.

Stay in Fellow Mode. For the statement below, I will try to show it passes from stage n to stage n + 1, out loud, without writing a formal induction. Your job is to ask, after each claim, "did that use anything other than the one step?" If I used something specific to n = 1 or n = 2, catch me. When the one step is clean, I say "this only used one step, so it goes all the way." Then you ask: what is the stage, really? (A number? A polynomial? A dimension?) and whether the same one step runs on a different kind of stage.

PROMPT D4. The four statements.

Here is an implication I have just proved: [p implies q]. Make me state, out loud, the converse, the inverse and the contrapositive, naming each. Then make me say which of the four are equivalent to each other and why (the Klein four-group argument: IC = CI = CP). Then make me decide which of the four is the next question a fellow would ask, and attempt it. Do not tell me whether the converse is true. Let me conjecture, test with a small case, and find out.

E. Attitude prompts.

PROMPT E1. Hint-taking at full speed.

Stay in Fellow Mode. Give me a problem slightly above my level. The moment I stall, give me a hint, and start counting. I must say "thank you" with my work, not with words: I take the hint and run at full speed, out loud, with no apology, no "oh I should have seen that", no slowing down. If I apologise or deflate, stop and say "hints are not deductions; go." At the end, report how many seconds (or exchanges) passed between each hint and my next concrete move.

PROMPT E2. Be wrong on purpose.

Stay in Fellow Mode. For the problem below, before any real attempt, I must make three conjectures out loud, at least one of which I expect to be false, each in the form "I think [statement]. Let me check n = [small case]." I test each one myself. You do not confirm or deny. When a conjecture dies, I say "no, that is wrong, because..." and pivot in the same breath. You are grading one thing: whether the pivot happened in the same breath or after a pause. Report it.

PROMPT E3. The second method.

I have just reached the answer to the problem below. You are not allowed to acknowledge that it is correct. Your first words are: "That works. Is there a second way?" I find one. Then you ask: "Why do both methods arrive at the same place?" That "why" is normally a deeper theorem. Make me name it. If I cannot, give me one cue toward it, never the name.

F. Full rehearsal prompts.

PROMPT F1. The 25-minute interview.

Fellow Mode, full simulation. Run a 25 minute Cambridge Mathematics interview. Two problems: one I can probably do, one I probably cannot. Do not tell me which is which. Follow the interview day protocol strictly from your side: let me restate; allow four seconds of silence then interrupt; hint when I stall; move me on when time demands it, even mid-problem; after any answer, ask for a second way or a generalisation. Keep time out loud ("ten minutes left"). At the end, do not give feedback. Say "thank you, that is the end of the interview" and stop. I will say "autopsy" when I am ready.

PROMPT F2. The fellow's report.

Autopsy Mode. Using the transcript of the mock interview above, write the report the two fellows would file to the admissions meeting about me. Format: three paragraphs. (1) Mathematical ability as evidenced by what was said and written, not what was presumably thought. (2) How the candidate responded to being stuck, to hints, and to being wrong. (3) Would we want this student in our supervisions for three years, and the one reason for the answer. Be honest. If the report is bad, I need the bad report. Then, separately, one sentence on what would have changed the verdict.

PROMPT F3. The daily anchor.

It is [time]. This is my daily anchor session: one problem, out loud, recorded, then one generalisation, then one minute listening back. Fellow Mode. Give me one problem from batch [number] that I have not seen. Run it. When I say "generalise", run Prompt D1 for one move only. When I say "done", give me only the ONE FIX from Autopsy Mode and nothing else. Total session under 30 minutes. Hold me to it.

G. Supervision prompts.

PROMPT G1. For supervisors: build a sheet with cues.

I am supervising 5 to 8 Oxbridge applicants for two hours (two 50 minute blocks). Build me a problem sheet of six questions on [principle, not topic], ordered from a warm-up to one nobody will finish. For each question give me: the problem; the single cue I should whisper if the room stalls (as a question, never a step); the generalisation I should ask for at the end; and the one wrong turn to expect. Do not give me solutions. I will solve them myself before the session, out loud, as my own rep.

PROMPT G2. For students: before the supervision.

Fellow Mode. Here is the sheet my supervisor set (paste). For each question, I will attempt it out loud and then write my one-sentence stuck point in the form from Prompt C4. You do not help me solve anything. Your only job is to make sure every stuck point is precise and every attempt was spoken, not silent. At the end, list my six stuck points so I can bring them to the room.

PROMPT G3. After the supervision: the recall.

Autopsy Mode. I have just finished a two hour supervision. I will now recall, out loud, every statement that was made in the room, including the wrong ones, in order, without notes. You will transcribe them as a numbered list as I go. When I stop, ask: which blocks could have been truncated? How did each leap happen, and what was the cue? Which theorem is each leap, really? Then ask me for the one thing I will say differently next time. Then stop.

Part III: The question batches.

Twelve batches, filed by principle. Each question has a **cue** (the thing in the question that should fire a circuit) and a **generalise** line (the move to attempt after the answer). No solutions anywhere in this document. Run every question through Prompt 0 and end with Prompt D1. If you solve a batch silently on paper, you have done TMUA preparation, not interview preparation.

Difficulty is interview level: Trinity specimen, STEP I, MAT long questions, BMO1 openers, and the classics every fellow has in the drawer. You will meet none of these in the room. You will meet their cues.

Batch 1. Counting with structure.

1.1 How many integers in [1, 6000] are multiples of none of 2, 3, 5? Now do [1, 5998]. **Cue:** 30 divides 6000; period. **Generalise:** any set of primes; what happens to the density as you add primes?

1.2 How many zeros does 100! end in? **Cue:** a trailing zero is a factor of 10; which prime is scarce? **Generalise:** in base 12? In which base does 100! end in the most zeros?

1.3 In how many ways can 2024 be written as a sum of two or more consecutive positive integers? **Cue:** the sum of k consecutive integers has a closed form; parity of k. **Generalise:** which numbers can be written in no way at all?

1.4 How many squares, of all sizes, are on an 8 by 8 chessboard? **Cue:** a k by k square is fixed by its top-left corner. **Generalise:** rectangles; an n by m board; cubes in an n by n by n cube.

1.5 A positive integer has exactly 12 positive divisors. What is the smallest such integer? **Cue:** divisor count from the prime factorisation. **Generalise:** which integers have an odd number of divisors, and why?

1.6 How many integers from 1 to 999 have digit sum exactly 10? **Cue:** three digits, each at most 9, summing to 10. **Generalise:** digit sum s; is there a symmetry between s and 27 − s?

Batch 2. Congruences and polynomials.

2.1 Show that if $a \equiv b \pmod m$ then $p(a) \equiv p(b) \pmod m$ for every polynomial p with integer coefficients. **Cue:** $a - b$ divides $a^k - b^k$. **Generalise:** is the converse true? For which p?

2.2 Show there is no polynomial p with integer coefficients such that $p(1) = 2$ and $p(3) = 5$. **Cue:** 2.1 with specific numbers. **Generalise:** what pairs of values at 1 and 3 are possible?

2.3 Show that $n^5 - n$ is divisible by 30 for every integer n. **Cue:** factorise; $30 = 2 \times 3 \times 5$. **Generalise:** $n^p - n$ and p; for which k is $n^k - n$ always divisible by k?

2.4 Find the last two digits of 7^{2024}. **Cue:** $7^4 = 2401$. **Generalise:** the last two digits of 7^n for every n; what is the period and why?

2.5 Find all primes p such that $p^2 + 2$ is also prime. **Cue:** squares modulo 3. **Generalise:** for which n can p, p + 2, p + 4 all be prime?

2.6 A polynomial with integer coefficients takes the value 7 at four distinct integers. Show it never takes the value 14. **Cue:** p(x) − 7 has four known factors. **Generalise:** replace 7 with any prime and 14 with twice it; does the argument survive? Replace 4 with 3?

Batch 3. Invariants and parity.

3.1 The numbers 1 to 100 are written on a board. You repeatedly erase two numbers a, b and write a + b − 1. After 99 steps one number remains. What is it? **Cue:** what does one step do to the sum and to the count? **Generalise:** replace a + b − 1 with ab + a + b.

3.2 Remove two opposite corners from a chessboard. Can the rest be tiled with 2 by 1 dominoes? **Cue:** colour. **Generalise:** remove any two squares of opposite colour; is it always tileable? Converse?

3.3 Seven cups stand upside down. In one move you turn over exactly four cups. Can you get all seven the right way up? **Cue:** what does one move do to the number of upside-down cups, modulo 2? **Generalise:** n cups, flip k at a time.

3.4 A knight on a chessboard returns to its starting square after n moves. Show n is even. **Cue:** the colour of the square changes every move. **Generalise:** what property of the board did this use? State it without chess.

3.5 Write the numbers 1, 2, ..., 10 in a row and insert + or − between them. Can the result be 0? **Cue:** start with all plus signs; what does changing one sign do? **Generalise:** for which n is it possible with 1 to n?

3.6 2024 lamps are off. For each k from 1 to 2024, toggle every lamp whose number is a multiple of k. Which lamps end on? **Cue:** lamp n is toggled once per divisor. **Generalise:** 1.5 again; which lamps are toggled exactly twice?

Batch 4. Small cases and extreme cases.

4.1 A function f on the positive reals satisfies $f(x)f(y) = f(x + y)$ and $f(1) = 2$. Find f(n) for positive integers n, then f(1/2), then f(p/q). **Cue:** n = 2 first; then what is f(1/2) squared? **Generalise:** what extra condition pins down f on all reals?

4.2 Into how many regions do n lines in general position divide the plane? **Cue:** n = 0, 1, 2, 3; what does the n-th line add? **Generalise:** circles instead of lines; planes in three dimensions.

4.3 Place n points on a circle and join every pair. Into how many regions is the disc divided (no three chords concurrent)? Compute n = 1 to 6. **Cue:** the pattern you see at n = 5 is a trap. **Generalise:** what did the small cases fool you into believing, and what should the lesson be about overfitting thin data?

4.4 For which n can a square be cut into exactly n smaller squares (not necessarily equal)? **Cue:** n = 4; then how do you get from n to n + 3? **Generalise:** which n are impossible, and can you prove it for n = 5?

4.5 If $x + 1/x = 3$, find $x^5 + 1/x^5$. **Cue:** let $a_n = x^n + 1/x^n$; relate a_{n+1} to a_n and a_{n-1}. **Generalise:** is a_n always an integer? For which x is a_n an integer for all n?

4.6 From the set {1, 2, ..., 2n} choose n + 1 numbers. Show two of them are coprime, and show one divides another. **Cue:** consecutive integers; odd parts. **Generalise:** is n + 1 sharp? Find n numbers with neither property.

Batch 5. Functions, iteration and fixed points.

5.1 Let $f(x) = 1/(1 - x)$. Compute f(f(x)) and f(f(f(x))). **Cue:** small cases, in the sense of iterating. **Generalise:** which other simple functions have $f^{\circ 3}$ = identity? $f^{\circ 2}$?

5.2 Show that every continuous function from [0, 1] to [0, 1] has a fixed point. **Cue:** consider g(x) = f(x) − x at the two ends. **Generalise:** does it hold on (0, 1)? On the whole real line? What is the converse?

5.3 Let $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{(2 + x_n)}$. Show the sequence is increasing and bounded, and find its limit. **Cue:** if it converges to L, what equation does L satisfy? **Generalise:** $\sqrt{(a + x_n)}$; for which a does it converge, and to what?

5.4 Find a function f from the reals to the reals, not the identity, with f(f(x)) = x for all x. Show any such f is a bijection. Can it be continuous and strictly increasing? **Cue:** what does f(f(x)) = x say about injectivity? **Generalise:** f(f(f(x))) = x.

5.5 Sketch $y = x^x$ for x > 0. Where is the minimum? What happens as x tends to 0 from above? **Cue:** take logs. **Generalise:** $y = x^{1/x}$; where is its maximum, and what does that tell you about $n^{1/n}$ for integers?

5.6 Let s(n) be the number of 1s in the binary expansion of n. Show s(2n) = s(n) and s(2n + 1) = s(n) + 1. Find the sum of s(n) over $0 \le n < 2^k$. **Cue:** one step: what does doubling do to the binary string? **Generalise:** base 3; the average number of 1s.

Batch 6. Graph sketching.

6.1 Sketch $y = \sin(x)/x$. What happens at $x = 0$? What curves bound it? **Cue:** the envelope $\pm 1/x$. **Generalise:** $\sin(x)/x^2$; $\sin(x^2)/x$.

6.2 Sketch $y = e^{-x^2}$ and $y = x\,e^{-x^2}$ on the same axes. **Cue:** even and odd; where is the second one stationary? **Generalise:** $x^n\,e^{-x^2}$; how many turning points?

6.3 Sketch $y = x^3 - 3x + c$ for $c = -3, -2, 0, 2, 3$. How many real roots in each case? **Cue:** the turning points do not move when c changes. **Generalise:** for $x^3 + px + q$, find the condition on p and q for three real roots.

6.4 Sketch $y = \ln(x)/x$. Where is the maximum? Use the sketch to decide which is larger: e^π or π^e. **Cue:** a^b vs b^a is $\ln(a)/a$ vs $\ln(b)/b$. **Generalise:** for which pairs of positive integers is $a^b = b^a$?

6.5 Sketch the curve $y^2 = x^2(x + 1)$. **Cue:** symmetry in y; what happens at the origin? **Generalise:** $y^2 = x^2(x + a)$ as a varies through 0 and negative values.

6.6 Sketch $y = |x| + |x - 1| + |x - 2|$. Where is the minimum? **Cue:** piecewise linear; the slope changes at three points. **Generalise:** $|x - a_1| + \ldots + |x - a_n|$; where is the minimum for even n, for odd n, and what statistic is that?

Batch 7. Inequalities and bounding.

7.1 Show $a^2 + b^2 \geq 2ab$ for all real a, b. Deduce the AM-GM inequality for two positive numbers. Then show $a^3 + b^3 + c^3 \geq 3abc$ for positive a, b, c. **Cue:** a square is non-negative; factorise $a^3 + b^3 + c^3 - 3abc$. **Generalise:** n numbers; when is equality?

7.2 Which is larger, 2^{100} or 10^{30}? No calculator. **Cue:** 2^{10} is close to 10^3; which side of it? **Generalise:** bound $\log_{10} 2$ from both sides using only integer powers.

7.3 Show that $1 + 1/2 + 1/3 + \ldots + 1/n > \ln(n + 1)$. **Cue:** draw the rectangles and the curve $1/x$. **Generalise:** an upper bound of the same form; does the difference converge?

7.4 Show that for $x > 0$, $e^x > 1 + x + x^2/2$. **Cue:** both sides agree at $x = 0$; compare derivatives. **Generalise:** add more terms; what is the general statement and does it hold for negative x?

7.5 Minimise $x + y$ subject to $xy = 100$ with $x, y > 0$. Then minimise $x + 2y$ under the same constraint. **Cue:** AM-GM, with a rescaling for the second. **Generalise:** minimise $ax + by$ subject to $xy = c$.

7.6 Show that $\sqrt{n} + \sqrt{(n + 2)} < 2\sqrt{(n + 1)}$ for every positive n. **Cue:** square both sides; or draw $\sqrt{x}$ and look at its shape. **Generalise:** $f(a) + f(b)$ vs $2f((a + b)/2)$ for which f? Name the property.

Batch 8. Algorithms and puzzles.

8.1 Twelve coins, one of which is heavier. Using a balance, what is the minimum number of weighings that always finds it? **Cue:** each weighing has three outcomes. **Generalise:** n coins; what if you do not know whether the fake is heavier or lighter?

8.2 Tower of Hanoi with n discs. Find the minimum number of moves. **Cue:** to move the biggest disc, where must the others be? One step: T(n) in terms of T(n − 1). **Generalise:** four pegs; is your recurrence still forced?

8.3 You have n numbers. How many comparisons are needed to find the largest? To find both the largest and the smallest? **Cue:** compare in pairs first. **Generalise:** the second largest; what information does the tournament already contain?

8.4 n people stand in a circle numbered 1 to n. Starting from 1, every second person is removed until one remains. Who survives? Compute for n = 1 to 16. **Cue:** what happens when n is a power of 2; what does adding one person do? **Generalise:** write n in binary and describe the survivor.

8.5 A frog starts at 0 on the integer line and jumps +1 or −1 each second. In how many ways can it be at position k after n jumps? **Cue:** count the +1s; parity of n and k. **Generalise:** probability of returning to 0 after 2n jumps; what happens to it as n grows?

8.6 You have two identical eggs and a 100-floor building. An egg breaks if dropped from floor F or above. Minimise the worst-case number of drops to find F. **Cue:** after the first egg breaks you must go one floor at a time; so the gaps should shrink. **Generalise:** three eggs; n floors.

Batch 9. Geometry, with everything named.

9.1 Triangle ABC has AB = AC. The perpendicular from A meets BC at D. Prove BD = DC, naming every object and every congruence. **Cue:** two triangles sharing AD. **Generalise:** state and prove the converse.

9.2 P is a point inside an equilateral triangle. Show the sum of the distances from P to the three sides is constant. **Cue:** join P to the vertices and think about area. **Generalise:** a regular n-gon; what if P is outside?

9.3 Prove that the angle in a semicircle is a right angle. **Cue:** join the centre; two isosceles triangles. **Generalise:** the angle at the centre is twice the angle at the circumference; and the converse: the locus of points seeing a fixed segment at a right angle.

9.4 A and B are fixed points. Find the locus of points P with PA = 2PB. **Cue:** coordinates, with A and B placed cleverly. **Generalise:** PA = kPB for general k; what happens at k = 1 and as k tends to 0 or infinity?

9.5 Find the rectangle of largest area inscribed in a circle of radius r. Then in a semicircle of radius r. **Cue:** symmetry, then AM–GM or calculus. **Generalise:** inscribed in an ellipse; a box in a sphere.

9.6 Two circles meet at A and B. A line through A meets the circles again at P and Q. Show that the angle PBQ does not depend on the line. **Cue:** angles in the same segment, in each circle separately. **Generalise:** what happens as the line through A becomes tangent to one circle?

Batch 10. Sequences and recurrences.

10.1 With $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$, show $F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$. **Cue:** write each F_k as a difference; telescope. **Generalise:** the sum of the even-indexed terms; the sum of squares.

10.2 $a_1 = 1$ and $a_{n+1} = 2a_n + 1$. Find a closed form. **Cue:** what does $a_n + 1$ do? **Generalise:** $a_{n+1} = p\,a_n + q$; find the fixed point first.

10.3 How many binary strings of length n contain no two consecutive 1s? **Cue:** split on the last digit. **Generalise:** no three consecutive 1s; strings over {0, 1, 2} with no two consecutive equal digits.

10.4 Show that $a_n = (1 + 1/n)^n$ is increasing and bounded above by 3. **Cue:** expand binomially and compare term by term; or AM-GM on n copies of $(1 + 1/n)$ and one 1. **Generalise:** $(1 + x/n)^n$ for fixed x.

10.5 Sum $1/(k(k + 1))$ from $k = 1$ to n. Then sum $1/(k(k + 1)(k + 2))$. **Cue:** partial fractions; telescoping. **Generalise:** the product of $(1 - 1/k^2)$ from $k = 2$ to n; does it telescope too?

10.6 $a_1 = 1$ and $a_{n+1} = a_n + 1/a_n$. Show a_n tends to infinity, and show $a_n^2 > 2n$ for $n \geq 2$. **Cue:** square the recurrence; telescope. **Generalise:** how good is $a_n \approx \sqrt{(2n)}$? Bound a_n^2 above as well.

Batch 11. Probability and expectation.

11.1 Roll a fair die until a 6 appears. What is the expected number of rolls? **Cue:** after a non-6, you are back where you started. **Generalise:** expected rolls until every face has appeared.

11.2 Three doors, one prize. You pick one; the host, who knows, opens a different door with no prize; you may switch. Should you? **Cue:** what did the host's choice tell you, and when did he have no choice? **Generalise:** n doors, host opens k; a host who does not know.

11.3 A random permutation of 1 to n. What is the expected number of fixed points? **Cue:** indicator for each position; linearity. **Generalise:** the variance; the probability of no fixed points as n grows.

11.4 Flip a fair coin until two heads in a row appear. What is the expected number of flips? **Cue:** states: no progress, one head. **Generalise:** until HT appears; why is the answer different from HH?

11.5 A stick is broken at a uniformly random point. What is the expected length of the longer piece? **Cue:** symmetry about the midpoint. **Generalise:** two random breaks; probability the three pieces form a triangle.

11.6 How many people are needed in a room for the probability that two share a birthday to exceed one half? **Cue:** compute the probability that all are different; approximate $1 - x$ by e^{-x}. **Generalise:** d days instead of 365; how does the answer scale with d?

Batch 12. Logic and proof structure.

12.1 Prove $\sqrt{2}$ is irrational. Then $\sqrt{3}$. Then $\sqrt{6}$. Then try the same proof on $\sqrt{4}$ and find exactly which line fails. **Cue:** the boundary is the theorem. **Generalise:** for which n is $\sqrt{n}$ irrational, in one sentence?

12.2 Prove: if n^2 is even then n is even. State the converse, the inverse and the contrapositive, and say which are true and which are equivalent to which. **Cue:** the Klein four-group of statements. **Generalise:** replace "even" with "divisible by 3", then "divisible by 4".

12.3 Prove there are infinitely many primes. Then prove there are infinitely many primes of the form $4k + 3$. **Cue:** a product of known primes, adjusted so it is forced to have a new factor of the right form. **Generalise:** try $4k + 1$ with the same trick; where exactly does it break?

12.4 Show there exist irrational numbers a and b with a^b rational. **Cue:** consider $\sqrt{2}^{\sqrt{2}}$; you do not need to know whether it is rational. **Generalise:** give a constructive example using a logarithm; what is the difference between the two proofs?

12.5 Five points lie in a unit square. Show two of them are at most $\sqrt{2}/2$ apart. **Cue:** four boxes, five pigeons. **Generalise:** n points in the square; the same question in an equilateral triangle.

12.6 Show that for every n there are n consecutive composite numbers. **Cue:** a number with many small factors, then add 2, 3, 4, ... **Generalise:** what does this say about gaps between primes, and what is the (open) converse question about small gaps?

How to run the batches.

1. One batch per week, one question per daily anchor (Prompt F3), the sixth question on the weekend with the full 25 minute rehearsal (Prompt F1 and F2).
2. Every question: Prompt 0, out loud, recorded. Then "generalise" (Prompt D1, one move). Then "autopsy" (Prompt 0b). Then the ONE FIX into your journal under the principle, not the topic.

3. Every batch: after the sixth question, run Prompt B3 on the circuit you extracted most often that week. Five new problems, same cue, different costume.
4. Every supervision: bring your six stuck points (Prompt G2). After it, run Prompt G3 before you sleep.
5. Twelve weeks covers all twelve batches. If you have fewer weeks, do not do fewer questions per batch; do fewer batches. Depth over count, always.

Common pitfalls.

- Pasting the question into the LLM without Prompt 0 first. It will solve it for you in one message and the question is gone.
- Typing the attempt silently and tidily. You are training writing, not speaking. Dictate, or use voice.
- Skipping the autopsy because the problem went well. A problem that went well still had a cue you noticed late and a sentence you said badly.
- Running a batch as a worksheet, six questions in an hour, no generalisation. That is a TMUA session wearing an interview costume.
- Treating the model's "yes, correct" as the end. It is not a fellow. Ask it to find the hole in your proof; there is often one.
- Collecting the batches and not running them. The Greenprint on the desk unopened is Redprint mistake 21.

Notes.

Everything referenced here is free at tmua9.com: the Interview Blueprint and Redprint in the Galleria, the Common Lectures and mock interviews on the TMUA9 and Polymatique YouTube channels, Polymatique supervisions at polymatique.com, and the Discord where you can be wrong in public. None of it is AI slop; the prompts above are how to use AI without it becoming slop. Every PDF other than NOIR PREMIERE is free and will always stay free.

You do not need to be a genius to run any of this. You need a phone with a voice memo app, one of these twelve batches, Prompt 0, and the willingness to hear yourself be bad at it for two weeks. I am rooting for you from afar. May God be with you.

"But as for you, be strong and do not give up, for your work will be rewarded."
2 Chronicles 15:7 NIV

Best, Jamie. TMUA9.

9.